Key

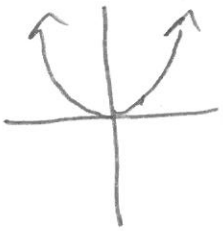
Investigating: Even and Odd Functions

Objective: Identify a function as even, odd or neither.

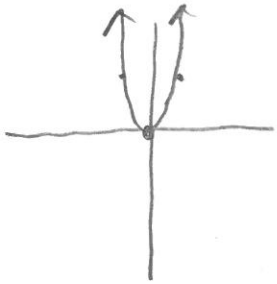
Investigating: Even and Odd Functions

Use your graphing calculator to graph each of the following functions. Draw a sketch of the graph under its equation.

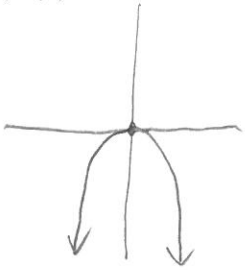
a) $f(x) = x^4$



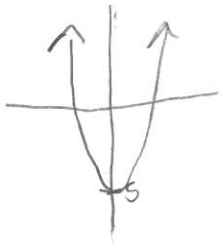
b) $f(x) = 2x^4 + 2x^2$



c) $f(x) = -3x^6 - 2x^4$



d) $f(x) = 2x^2 - 5$



All of the above functions are called **EVEN** functions.

a) What type of symmetry does each graph have? y axis

b) What is special about the exponents of each term in the functions? all even

c) What happens when you evaluate f(-x) for each of the functions?

$f(-x) = (-x)^4 = x^4$

$f(-x) = 2(-x)^4 + 2(-x)^2 = 2x^4 + 2x^2$

$f(-x) = -3(-x)^6 - 2(-x)^4 = -3x^6 - 2x^4$

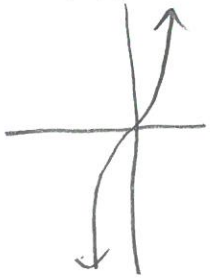
$f(-x) = 2(-x)^2 - 5 = 2x^2 - 5$

returns to original function

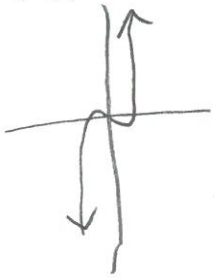
$f(-x) = f(x)$

Use your graphing calculator to graph each of the following functions. Draw a sketch of the graph under its equation.

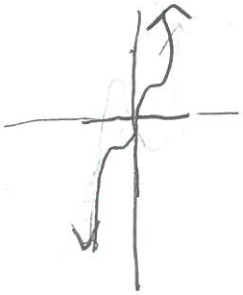
a) $f(x) = x^3$



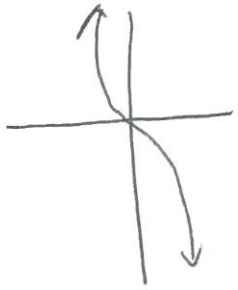
b) $f(x) = x^3 - x$



c) $f(x) = x^5 - 3x^3 + 4x$



d) $f(x) = -2x^3 - 5x$



All of the above functions are called **ODD** functions.

a) What type of symmetry does each graph have? origion

b) What is special about the exponents of each term in the equations?

ODD

c) What happens when you evaluate f(-x) for each of the functions?

$f(-x) = (-x)^3 = -x^3$

$f(-x) = (-x)^3 - (-x) = -x^3 + x$

$f(-x) = (-x)^5 + 3(-x)^3 + 4(-x) = -x^5 - 3x^3 - 4x$

$f(-x) = -2(-x)^3 - 5(-x) = 2x^3 + 5x$

all signs change

opposite $f(-x) = f(x)$

Key

In general,

An **EVEN** function has the following properties:

i) Its graph is symmetric about the y axis

ii) The exponents of all terms in its equation are even

iii) f(-x) = f(x)

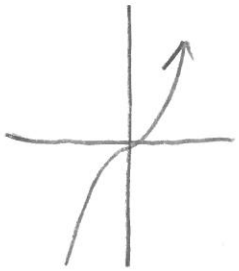
An **ODD** function has the following properties:

i) Its graph is symmetric about the origin

ii) The exponents of all terms in its equation are ODD

iii) f(-x) = -f(x)

Consider the function $f(x) = x^3 - x^2 + x$, would it be even or odd? Investigate all three properties from above.

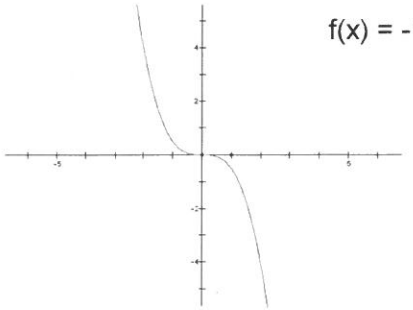


not symmetric
not all even or ODD
$f(-x) = (-x)^3 - (-x)^2 + (-x)$
$= -x^3 - x^2 - x$
not all signs change

Such a function is **NEITHER** even nor odd.

Example:

Determine whether the following is even, odd or neither. You must justify your answer by discussing all three properties.



$f(x) = -1/2x^3$

symmetric w/origin,
ODD exponent
$f(-x) = -\frac{1}{2}(-x)^3 = \frac{1}{2}x^3$
sign changes
$f(-x) = -f(x)$

Investigating: Even and Odd Functions - Worksheet

Determine whether each of the following is even, odd or neither. You must justify your answer by discussing all three properties.

a) $f(x) = -1/2x^4 + 3x^2$

even

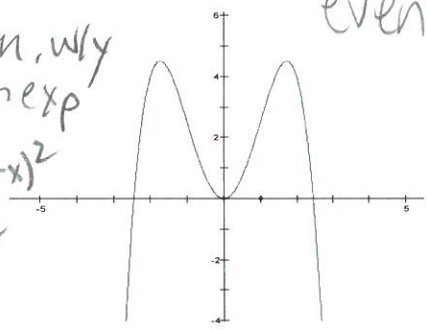
Symm. w/y

even exp

$-\frac{1}{2}(-x)^4+3(-x)^2$

$-\frac{1}{2}x^4+3x^2$

stays same



b) $f(x) = -x^2 + x$

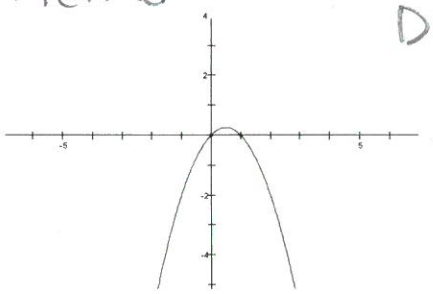
neither

no symmetry

Different exponents

$f(-x) = -(-x)^2+(-x)$

$= -x^2 - x$

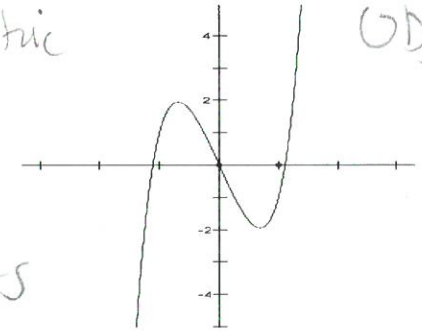


c) $f(x) = x^5 + 2x^3 - 4x$

symmetric w/ origin

(ODD)

all ODD exponents

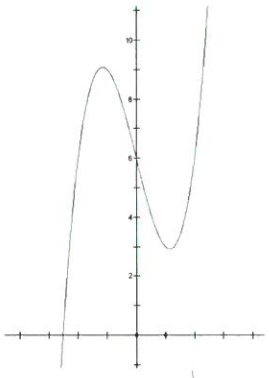


$f(-x) = (-x)^5+2(-x)^3-4(-x)$

$= -x^5 - 2x^3 + 4x$

all signs change

d) $f(x) = x^3 - 4x + 6$



neither

no symmetry w/y or origin

exponents ODD except 6

$f(-x) = (-x)^3-4(-x)+6$

$= -x^3+4x+6$

not all signs change

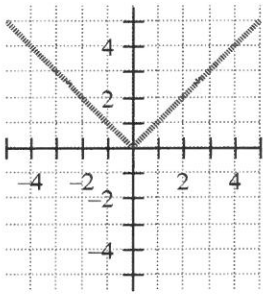
Symmetry

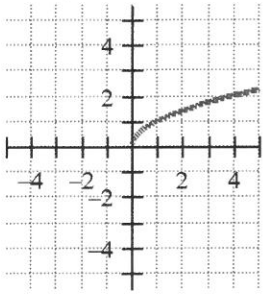
Name: Key

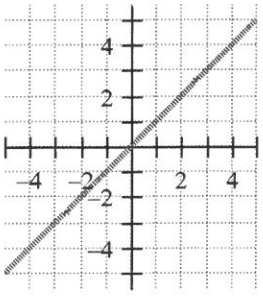
1. If a function is even, its graph is symmetric with respect to the yaxis.
   This also means that $f(-x) = f(x)$

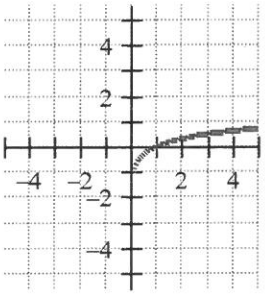
2. If a function is odd, its graph is symmetric with respect to the origin.
   This also means that $f(-x) = -f(x)$

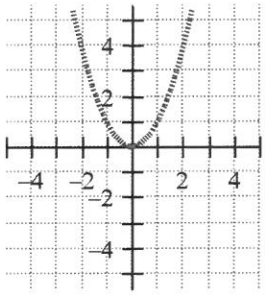
Determine whether each function graphed is even, odd, or neither

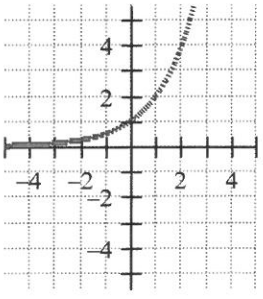
E 3. 

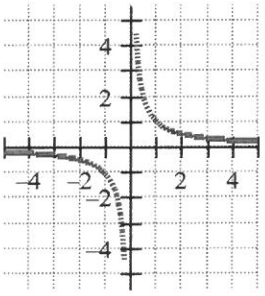
N 4. 

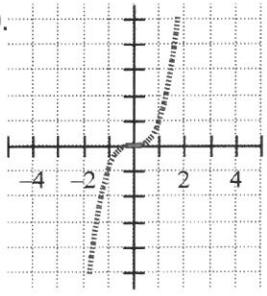
O 5. 

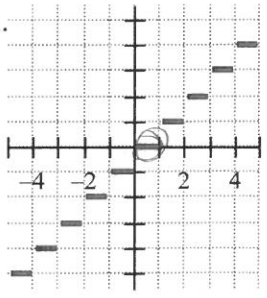
N 6. 

E 7. 

N 8. 

O 9. 

O 10. 

N 11. 

Determine algebraically whether each of the following functions is even, odd or neither.

12. $f(x)=4x+5$ N

$f(-x)=4(-x)+5$

$=-4x+5$

13. $f(x)=x^3-x$ O

$f(-x)=(-x)^3-(-x)$

$=-x^3+x$

14. $f(x)=x^2-6$ e

$f(-x)=(-x)^2-6$

$=x^2-6$

15. $f(x)=x^3-x-2$ N

$f(-x)=-(-x)^3-(-x)-2$

$=-x^3+x-2$

16. f(x)=|x| e

$f(-x)=|-x|$

$=x$

$f(x)=|x| \quad =x$

$=-x$

$f(-x)=|-x| \quad =x$

$=-x$

17. $f(x)=\frac{x^3-x}{x^5}$ Even

$f(-x)=\frac{(-x)^3-(-x)}{(-x)^5}=\frac{-x^3+x}{-x^5}$

$-\frac{(x^3-x)}{-x^5}$

$f(-x)=\frac{x^3-x}{x^5}=f(x)$

18. $f(x)=(x-4)^2$ N

$f(x)=x^2-8x+16$

$f(-x)=(-x)^2-8(-x)+16$

$=x^2+8x+16$

19. $f(x)=x^4-x^2+4$ E

$f(-x)=(-x)^4-(-x)^2+4$

$=x^4-x^2+4$